

Three-electron spin states and entanglement states

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Abstract

In this paper, we have given the symmetrical and antisymmetrical spin and space wave functions of three-electron, and further given the full total entanglement states for the three-electron, which are related to their space and spin wave function. When we study particles entanglement we not only consider their spin entanglement and also consider their space entanglement. Otherwise, we find that electrons entanglement are restricted to the teeny range, when electrons exceed the space range, their entanglement should be broken down even disappearance, which is accordance with the experiments results.

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1 Introduction

The quantum many-body problem is one of the most fascinating topics in modern physics, and as well as one of the most challenging [1]. The challenges stem from the Hilbert space growing exponentially with the magnitude of the systems if they are treated exactly. Fortunately, we do not need to address the entire Hilbert space, because the physical properties of the quantum many-body systems are determined by the ground state and some low excitation levels [2].

Entanglement is one of the most fundamentally nonclassical features of quantum mechanics and as such is highly important to the foundations of modern physics, e.g., in high energy physics [3, 4]. In a more practical view, entanglement is most relevant in two distinct respects: technological applications (such as e.g. quantum cryptography [5, 6] or future quantum computers [7]).

In recent years, frustrated spin systems have received a lot of attention due to its association with high-Tc super- conductivity [8], and the discovery of exotic frustrated phases [9]. There have been several attempts to characterize and study frustrated systems using quantum information concepts, which include entanglement area law [10, 11], fidelity [12], and quantum discord [13]. Various aspects of entanglement have also been experimentally investigated in frustrated quantum spin systems [14, 15].

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In this paper, we have given the symmetrical and antisymmetrical spin and space wave functions of three-electron, and further given the full total entanglement states for the three-electron, which are related to their space and spin wave function. When we study particles entanglement we not only consider their spin entanglement and also consider their space entanglement. Otherwise, we find that electrons entanglement are restricted to the teeny range, when electrons exceed the space range, their entanglement should be broken down even disappearance, which is accordance with the experiments results.

2 Two-electron spin states

In quantum mechanics, the total spin \vec{S} is the sum of three electrons spin \vec{s}_1 , \vec{s}_2 and \vec{s}_3

$$\vec{S}_{123} = \vec{s}_1 + \vec{s}_2 + \vec{s}_3 = \vec{S}_{12} + \vec{s}_3, \quad (1)$$

where

$$\vec{S}_{12} = \vec{s}_1 + \vec{s}_2, \quad (2)$$

and \vec{S}_{12}^2 , \vec{S}_{123}^2 are

$$\begin{aligned} \vec{S}_{12}^2 &= (\vec{s}_1 + \vec{s}_2)^2 \\ &= \vec{s}_1^2 + \vec{s}_2^2 + 2\vec{s}_1 \cdot \vec{s}_2 \\ &= \frac{3}{2} + 2\vec{s}_1 \cdot \vec{s}_2, \end{aligned} \quad (3)$$

$$\begin{aligned} \vec{S}_{123}^2 &= (\vec{s}_1 + \vec{s}_2 + \vec{s}_3)^2 \\ &= \vec{s}_1^2 + \vec{s}_2^2 + \vec{s}_3^2 + 2(\vec{s}_1 \cdot \vec{s}_2 + \vec{s}_2 \cdot \vec{s}_3 + \vec{s}_1 \cdot \vec{s}_3) \\ &= \frac{9}{4} + 2(\vec{s}_1 \cdot \vec{s}_2 + \vec{s}_2 \cdot \vec{s}_3 + \vec{s}_1 \cdot \vec{s}_3), \end{aligned} \quad (4)$$

Obviously, \vec{S}_{12}^2 commutes with \vec{S}_{12} and \vec{s}_3 , and \vec{S}_{12}^2 commutes with \vec{S}_{123} . So, \vec{S}_{12}^2 , \vec{S}_{123}^2 and $(\vec{S}_{123})_z$ commute with each other, and they have common eigenfunctions. The eigenvalues and quantum number of \vec{S}_{12}^2 and \vec{S}_{123}^2 are

$$\vec{S}_{12}^2 = S'(S' + 1), \quad S' = 0, 1, \quad (5)$$

$$\vec{S}_{123}^2 = S(S + 1), \quad S = \frac{1}{2}(S' = 0); \quad \frac{1}{2}, \frac{3}{2}(S' = 1). \quad (6)$$

For two-electron, \vec{S}_{12}^2 and $(\vec{S}_{12})_z$ have common spin eigenstates, they are:

$$\chi_{11}(1, 2) = \chi_{\frac{1}{2}}(s_{1z})\chi_{\frac{1}{2}}(s_{2z}), \quad (S' = 1, \quad (s_{12})_z = 1) \quad (7)$$

$$\chi_{10}(1, 2) = \frac{1}{\sqrt{2}}[\chi_{\frac{1}{2}}(s_{1z})\chi_{-\frac{1}{2}}(s_{2z}) + \chi_{-\frac{1}{2}}(s_{1z})\chi_{\frac{1}{2}}(s_{2z})], \quad (S' = 1, \quad (s_{12})_z = 0) \quad (8)$$

$$\chi_{1-1}(1, 2) = \chi_{-\frac{1}{2}}(s_{1z})\chi_{-\frac{1}{2}}(s_{2z}), \quad (S' = 1, \quad (s_{12})_z = -1) \quad (9)$$

$$\chi_{00}(1, 2) = \frac{1}{\sqrt{2}}[\chi_{\frac{1}{2}}(s_{1z})\chi_{-\frac{1}{2}}(s_{2z}) - \chi_{-\frac{1}{2}}(s_{1z})\chi_{\frac{1}{2}}(s_{2z})], \quad (S' = 0, \quad (s_{12})_z = 0) \quad (10)$$

where $\chi_{\frac{1}{2}}(s_{1z})$ and $\chi_{\frac{1}{2}}(s_{2z})$ are the spin eigenstates of s_{1z} and s_{2z} .
Defining single-electron lowering operators s_{i-} as

$$s_{i-} = s_{ix} - i s_{iy}, \quad (i = 1, 2, 3) \quad (11)$$

and the total lowering operator of three-electron is

$$S_{123-} = s_{1-} + s_{2-} + s_{3-}, \quad (12)$$

by the angular theory, we have

$$s_{-}|jm\rangle = \sqrt{(j+m)(j-m+1)}|jm-1\rangle. \quad (13)$$

By equation (13), we have

$$s_{1-}\chi_{\frac{1}{2}}(s_{1z}) = \chi_{-\frac{1}{2}}(s_{1z}), \quad s_{1-}\chi_{-\frac{1}{2}}(s_{1z}) = 0 \quad (14)$$

$$s_{2-}\chi_{\frac{1}{2}}(s_{2z}) = \chi_{-\frac{1}{2}}(s_{2z}), \quad s_{2-}\chi_{-\frac{1}{2}}(s_{2z}) = 0 \quad (15)$$

$$s_{3-}\chi_{\frac{1}{2}}(s_{3z}) = \chi_{-\frac{1}{2}}(s_{3z}), \quad s_{3-}\chi_{-\frac{1}{2}}(s_{3z}) = 0. \quad (16)$$

3 Three-electron spin states

In the following, we should give the spin states of three-electron. For two-electron, the states of $S' = 1$ are symmetrical. So, all the three-electron states of $S' = 1$ should be symmetrical. For two-electron, the states of $S' = 0$ are antisymmetrical. So, all the three-electron states of $S' = 0$ should be antisymmetrical. We should give all symmetrical and antisymmetrical states for three-electron.

(1) $S' = 1, S = \frac{3}{2}$ spin wave functions $\chi_{S'SM}$ of three-electron

(a) The spin wave function $\chi_{1\frac{3}{2}\frac{3}{2}}$

the three quantum number take maximum value, i.e., $S' = 1, S = \frac{3}{2}$ and $M = \frac{3}{2}$. It can be written directly

$$\begin{aligned} \chi_{1\frac{3}{2}\frac{3}{2}} &= \chi_{11}(1, 2)\chi_{\frac{1}{2}}(s_{3z}) \\ &= \chi_{\frac{1}{2}}(s_{1z})\chi_{\frac{1}{2}}(s_{2z})\chi_{\frac{1}{2}}(s_{3z}). \end{aligned} \quad (17)$$

The state $\chi_{1\frac{3}{2}\frac{3}{2}}$ is symmetrical when exchange electron 1, 2 and 3.

(b) The spin wave function $\chi_{1\frac{3}{2}\frac{1}{2}}$

By operating lowering operators S_{123-} on $\chi_{1\frac{3}{2}\frac{3}{2}}$, we can obtain $\chi_{1\frac{3}{2}\frac{1}{2}}$

$$\begin{aligned} S_{123-}\chi_{1\frac{3}{2}\frac{3}{2}} &= (s_{1-} + s_{2-} + s_{3-})\chi_{\frac{1}{2}}(s_{1z})\chi_{\frac{1}{2}}(s_{2z})\chi_{\frac{1}{2}}(s_{3z}) \\ &= (s_{1-}\chi_{\frac{1}{2}}(s_{1z}))\chi_{\frac{1}{2}}(s_{2z})\chi_{\frac{1}{2}}(s_{3z}) + \chi_{\frac{1}{2}}(s_{1z})(s_{2-}\chi_{\frac{1}{2}}(s_{2z}))\chi_{\frac{1}{2}}(s_{3z}) \\ &\quad + \chi_{\frac{1}{2}}(s_{1z})\chi_{\frac{1}{2}}(s_{2z})(s_{3-}\chi_{\frac{1}{2}}(s_{3z})) \\ &= \chi_{-\frac{1}{2}}(s_{1z})\chi_{\frac{1}{2}}(s_{2z})\chi_{\frac{1}{2}}(s_{3z}) + \chi_{\frac{1}{2}}(s_{1z})\chi_{-\frac{1}{2}}(s_{2z})\chi_{\frac{1}{2}}(s_{3z}) \\ &\quad + \chi_{\frac{1}{2}}(s_{1z})\chi_{\frac{1}{2}}(s_{2z})\chi_{-\frac{1}{2}}(s_{3z}), \end{aligned} \quad (18)$$

by equation (13), we have

$$S_{123-}\chi_{1\frac{3}{2}\frac{3}{2}} = \sqrt{3}\chi_{1\frac{3}{2}\frac{1}{2}}, \quad (19)$$

or

$$\begin{aligned}\chi_{1\frac{3}{2}\frac{1}{2}} &= \frac{1}{\sqrt{3}}[\chi_{-\frac{1}{2}}(s_{1z})\chi_{\frac{1}{2}}(s_{2z})\chi_{\frac{1}{2}}(s_{3z}) + \chi_{\frac{1}{2}}(s_{1z})\chi_{-\frac{1}{2}}(s_{2z})\chi_{\frac{1}{2}}(s_{3z}) \\ &+ \chi_{-\frac{1}{2}}(s_{1z})\chi_{\frac{1}{2}}(s_{2z})\chi_{-\frac{1}{2}}(s_{3z})].\end{aligned}\quad (20)$$

The state $\chi_{1\frac{3}{2}\frac{1}{2}}$ is symmetrical when exchange electron 1, 2 and 3.

(c) The spin wave function $\chi_{1\frac{3}{2}-\frac{1}{2}}$

By operating lowering operators S_{123-} on $\chi_{1\frac{3}{2}\frac{1}{2}}$, we can obtain

$$\begin{aligned}S_{123-}\chi_{1\frac{3}{2}\frac{1}{2}} &= \frac{1}{\sqrt{3}}[(s_1 - \chi_{-\frac{1}{2}}(s_{1z}))\chi_{\frac{1}{2}}(s_{2z})\chi_{\frac{1}{2}}(s_{3z}) + (s_1 - \chi_{\frac{1}{2}}(s_{1z}))\chi_{-\frac{1}{2}}(s_{2z})\chi_{\frac{1}{2}}(s_{3z}) \\ &+ (s_1 - \chi_{\frac{1}{2}}(s_{1z}))\chi_{\frac{1}{2}}(s_{2z})\chi_{-\frac{1}{2}}(s_{3z}) + \chi_{-\frac{1}{2}}(s_{1z})(s_2 - \chi_{\frac{1}{2}}(s_{2z}))\chi_{\frac{1}{2}}(s_{3z}) \\ &+ \chi_{\frac{1}{2}}(s_{1z})(s_2 - \chi_{-\frac{1}{2}}(s_{2z}))\chi_{\frac{1}{2}}(s_{3z}) + \chi_{\frac{1}{2}}(s_{1z})(s_2 - \chi_{\frac{1}{2}}(s_{2z}))\chi_{-\frac{1}{2}}(s_{3z}) \\ &+ \chi_{-\frac{1}{2}}(s_{1z})\chi_{\frac{1}{2}}(s_{2z})(s_3 - \chi_{\frac{1}{2}}(s_{3z})) + \chi_{\frac{1}{2}}(s_{1z})\chi_{-\frac{1}{2}}(s_{2z})(s_3 - \chi_{\frac{1}{2}}(s_{3z})) \\ &+ \chi_{\frac{1}{2}}(s_{1z})\chi_{\frac{1}{2}}(s_{2z})(s_3 - \chi_{-\frac{1}{2}}(s_{3z})) \\ &= \frac{1}{\sqrt{3}}[\chi_{-\frac{1}{2}}(s_{1z})\chi_{-\frac{1}{2}}(s_{2z})\chi_{\frac{1}{2}}(s_{3z}) + \chi_{-\frac{1}{2}}(s_{1z})\chi_{\frac{1}{2}}(s_{2z})\chi_{-\frac{1}{2}}(s_{3z}) \\ &+ \chi_{-\frac{1}{2}}(s_{1z})\chi_{-\frac{1}{2}}(s_{2z})\chi_{\frac{1}{2}}(s_{3z}) + \chi_{\frac{1}{2}}(s_{1z})\chi_{-\frac{1}{2}}(s_{2z})\chi_{-\frac{1}{2}}(s_{3z}) \\ &+ \chi_{-\frac{1}{2}}(s_{1z})\chi_{\frac{1}{2}}(s_{2z})\chi_{-\frac{1}{2}}(s_{3z}) + \chi_{\frac{1}{2}}(s_{1z})\chi_{-\frac{1}{2}}(s_{2z})\chi_{-\frac{1}{2}}(s_{3z})] \\ &= \frac{2}{\sqrt{3}}[\chi_{-\frac{1}{2}}(s_{1z})\chi_{-\frac{1}{2}}(s_{2z})\chi_{\frac{1}{2}}(s_{3z}) + \chi_{-\frac{1}{2}}(s_{1z})\chi_{\frac{1}{2}}(s_{2z})\chi_{-\frac{1}{2}}(s_{3z}) \\ &+ \chi_{\frac{1}{2}}(s_{1z})\chi_{-\frac{1}{2}}(s_{2z})\chi_{-\frac{1}{2}}(s_{3z})],\end{aligned}\quad (21)$$

by equation (13), we have

$$S_{123-}\chi_{1\frac{3}{2}\frac{1}{2}} = \sqrt{4}\chi_{1\frac{3}{2}-\frac{1}{2}},\quad (22)$$

or

$$\begin{aligned}\chi_{1\frac{3}{2}-\frac{1}{2}} &= \frac{1}{\sqrt{3}}[\chi_{-\frac{1}{2}}(s_{1z})\chi_{-\frac{1}{2}}(s_{2z})\chi_{\frac{1}{2}}(s_{3z}) + \chi_{-\frac{1}{2}}(s_{1z})\chi_{\frac{1}{2}}(s_{2z})\chi_{-\frac{1}{2}}(s_{3z}) \\ &+ \chi_{\frac{1}{2}}(s_{1z})\chi_{-\frac{1}{2}}(s_{2z})\chi_{-\frac{1}{2}}(s_{3z})].\end{aligned}\quad (23)$$

The state $\chi_{1\frac{3}{2}-\frac{1}{2}}$ is symmetrical when exchange electron 1, 2 and 3.

(d) The spin wave function $\chi_{1\frac{3}{2}-\frac{3}{2}}$

when quantum number M take minimum value, i.e., $M = -\frac{3}{2}$, the spin wave function of three-electron can be written directly

$$\chi_{1\frac{3}{2}-\frac{3}{2}} = \chi_{-\frac{1}{2}}(s_{1z})\chi_{-\frac{1}{2}}(s_{2z})\chi_{-\frac{1}{2}}(s_{3z}).\quad (24)$$

The state $\chi_{1\frac{3}{2}-\frac{3}{2}}$ is symmetrical when exchange electron 1, 2 and 3.

$$\chi_{1\frac{3}{2}\frac{1}{2}} = \frac{1}{\sqrt{3}}[\chi_{11}(1,2)\chi_{-\frac{1}{2}}(3) + \sqrt{2}\chi_{10}(1,2)\chi_{\frac{1}{2}}(3)],\quad (25)$$

(2) $S' = 1$, $S = \frac{1}{2}$ spin wave functions $\chi_{S'SM}$ of three-electron

(a) The spin wave function $\chi_{1\frac{1}{2}\frac{1}{2}}$

We firstly calculate $\chi_{1\frac{1}{2}\frac{1}{2}}$ state, it includes two $\chi_{\frac{1}{2}}$ states and one $\chi_{-\frac{1}{2}}$ state, and it is the linear superposition of $\chi_{11}(s_{1z}, s_{2z})\chi_{-\frac{1}{2}}(s_{3z})$ and $\chi_{10}(s_{1z}, s_{2z})\chi_{\frac{1}{2}}(s_{3z})$. Since $\chi_{1\frac{1}{2}\frac{1}{2}}$ and $\chi_{1\frac{3}{2}\frac{1}{2}}$ is orthogonal, and the $\chi_{1\frac{3}{2}\frac{1}{2}}$ state can be written as

$$\chi_{1\frac{3}{2}\frac{1}{2}} = \frac{1}{\sqrt{3}}[\chi_{11}(1,2)\chi_{-\frac{1}{2}}(3) + \sqrt{2}\chi_{10}(1,2)\chi_{\frac{1}{2}}(3)],\quad (26)$$

and state $\chi_{1\frac{1}{2}\frac{1}{2}}$ can be written as

$$\begin{aligned}
\chi_{1\frac{1}{2}\frac{1}{2}} &= \frac{1}{\sqrt{3}}[\sqrt{2}\chi_{11}(1,2)\chi_{-\frac{1}{2}}(3) - \chi_{10}(1,2)\chi_{\frac{1}{2}}(3)] \\
&= \frac{1}{\sqrt{6}}[2\chi_{\frac{1}{2}}(s_{1z})\chi_{\frac{1}{2}}(s_{2z})\chi_{-\frac{1}{2}}(s_{3z}) - \chi_{\frac{1}{2}}(s_{1z})\chi_{-\frac{1}{2}}(s_{2z})\chi_{\frac{1}{2}}(s_{3z}) \\
&\quad - \chi_{-\frac{1}{2}}(s_{1z})\chi_{\frac{1}{2}}(s_{2z})\chi_{\frac{1}{2}}(s_{3z})] \\
&= \chi_{1\frac{1}{2}\frac{1}{2}}^{S(12)}(123).
\end{aligned} \tag{27}$$

The state $\chi_{1\frac{1}{2}\frac{1}{2}}$ should be symmetrical, but equation (27) is symmetrical for exchanging photon 1 and 2, and not full symmetrical form. It should be symmetrization, and it is

$$\chi_{1\frac{1}{2}\frac{1}{2}}^S = \frac{1}{\sqrt{3}}[\chi_{1\frac{1}{2}\frac{1}{2}}^{S(12)}(123) + \chi_{1\frac{1}{2}\frac{1}{2}}^{S(13)}(123) + \chi_{1\frac{1}{2}\frac{1}{2}}^{S(23)}(123)]. \tag{28}$$

where $\chi_{1\frac{1}{2}\frac{1}{2}}^{S(12)}(123)$ is symmetrical for exchanging photon 1 and 2, $\chi_{1\frac{1}{2}\frac{1}{2}}^{S(13)}(123)$ is symmetrical for exchanging photon 1 and 3, and the state $\chi_{1\frac{1}{2}\frac{1}{2}}^{S(13)}(123)$ can be obtained by the state $\chi_{1\frac{1}{2}\frac{1}{2}}^{S(12)}(123)$ exchanging photon 2 and 3. Obviously, the state $\chi_{1\frac{1}{2}\frac{1}{2}}^S$ is full symmetrical for exchanging photon 1, 2 and 3.

(b) The spin wave function $\chi_{1\frac{1}{2}-\frac{1}{2}}$

Since state $\chi_{1\frac{3}{2}-\frac{1}{2}}$ and state $\chi_{1\frac{1}{2}-\frac{1}{2}}$ is orthogonal, and state $\chi_{1\frac{3}{2}-\frac{1}{2}}$ can be written as

$$\chi_{1\frac{3}{2}-\frac{1}{2}} = \frac{1}{\sqrt{3}}[\sqrt{2}\chi_{10}(1,2)\chi_{-\frac{1}{2}}(3) + \chi_{1-1}(1,2)\chi_{\frac{1}{2}}(3)], \tag{29}$$

and state $\chi_{1\frac{1}{2}-\frac{1}{2}}$ can be written as

$$\begin{aligned}
\chi_{1\frac{1}{2}-\frac{1}{2}} &= \frac{1}{\sqrt{3}}[\chi_{10}(1,2)\chi_{-\frac{1}{2}}(3) - \sqrt{2}\chi_{1-1}(1,2)\chi_{\frac{1}{2}}(3)] \\
&= \frac{1}{\sqrt{6}}[\chi_{\frac{1}{2}}(1)\chi_{-\frac{1}{2}}(2)\chi_{-\frac{1}{2}}(3) + \chi_{-\frac{1}{2}}(1)\chi_{\frac{1}{2}}(2)\chi_{-\frac{1}{2}}(3) \\
&\quad - 2\chi_{-\frac{1}{2}}(1)\chi_{-\frac{1}{2}}(2)\chi_{\frac{1}{2}}(3)] \\
&= \chi_{1\frac{1}{2}-\frac{1}{2}}^{S(12)}(123).
\end{aligned} \tag{30}$$

The state $\chi_{1\frac{1}{2}-\frac{1}{2}}$ should be symmetrical, but equation (30) is symmetrical for exchanging photon 1 and 2, and not full symmetrical form. It should be symmetrization, and it is

$$\chi_{1\frac{1}{2}-\frac{1}{2}}^S = \frac{1}{\sqrt{3}}[\chi_{1\frac{1}{2}-\frac{1}{2}}^{S(12)}(123) + \chi_{1\frac{1}{2}-\frac{1}{2}}^{S(13)}(123) + \chi_{1\frac{1}{2}-\frac{1}{2}}^{S(23)}(123)]. \tag{31}$$

(3) $S' = 0$, $S = \frac{1}{2}$ spin wave functions $\chi_{S'SM}$ of three-electron

(a) The spin wave function $\chi_{0\frac{1}{2}\frac{1}{2}}$

For $S' = 0$, the spin wave functions of electron 1 and 2 is $\chi_{00}(1,2)$. When $M = \frac{1}{2}$, the state $\chi_{0\frac{1}{2}\frac{1}{2}}$ can be written as directly

$$\begin{aligned}
\chi_{0\frac{1}{2}\frac{1}{2}} &= \chi_{00}(1,2)\chi_{\frac{1}{2}}(3) \\
&= \frac{1}{\sqrt{2}}[\chi_{\frac{1}{2}}(1)\chi_{-\frac{1}{2}}(2)\chi_{\frac{1}{2}}(3) - \chi_{-\frac{1}{2}}(1)\chi_{\frac{1}{2}}(2)\chi_{\frac{1}{2}}(3)] \\
&= \chi_{0\frac{1}{2}\frac{1}{2}}^{A(12)}(123).
\end{aligned} \tag{32}$$

The state $\chi_{0\frac{1}{2}\frac{1}{2}}$ should be antisymmetrical, but equation (32) is antisymmetrical for exchanging photon 1 and 2, and not full antisymmetrical form. It should be antisymmetrization, and it is

$$\chi_{0\frac{1}{2}\frac{1}{2}}^A = \frac{1}{\sqrt{3}}[\chi_{0\frac{1}{2}\frac{1}{2}}^{A(12)}(123) + \chi_{0\frac{1}{2}\frac{1}{2}}^{A(13)}(123) + \chi_{0\frac{1}{2}\frac{1}{2}}^{A(23)}(123)]. \tag{33}$$

(b) The spin wave function $\chi_{0\frac{1}{2}-\frac{1}{2}}$

When $M = \frac{1}{2}$, the state $\chi_{0\frac{1}{2}-\frac{1}{2}}$ can be also written as directly

$$\begin{aligned}\chi_{0\frac{1}{2}-\frac{1}{2}} &= \chi_{00}(1, 2)\chi_{-\frac{1}{2}}(3) \\ &= \frac{1}{\sqrt{2}}[\chi_{\frac{1}{2}}(1)\chi_{-\frac{1}{2}}(2)\chi_{-\frac{1}{2}}(3) - \chi_{-\frac{1}{2}}(1)\chi_{\frac{1}{2}}(2)\chi_{-\frac{1}{2}}(3)],\end{aligned}\quad (34)$$

The state $\chi_{0\frac{1}{2}-\frac{1}{2}}$ should be antisymmetrical, but equation (34) is antisymmetrical for exchanging photon 1 and 2, and not full antisymmetrical form. It should be antisymmetrization, and it is

$$\chi_{0\frac{1}{2}-\frac{1}{2}}^A = \frac{1}{\sqrt{3}}[\chi_{0\frac{1}{2}-\frac{1}{2}}^{A(12)}(123) + \chi_{0\frac{1}{2}-\frac{1}{2}}^{A(13)}(123) + \chi_{0\frac{1}{2}-\frac{1}{2}}^{A(23)}(123)].\quad (35)$$

4 Three-electron space states

When we neglect the interaction among electrons, the space wave functions of three-electron can be written as

(1) The symmetrical space wave functions of three-electron

$$\psi_1^S(\vec{r}_1, \vec{r}_2, \vec{r}_3) = \psi_n(\vec{r}_1)\psi_n(\vec{r}_2)\psi_n(\vec{r}_3), \quad (n = m = l) \quad (36)$$

$$\begin{aligned}\psi_2^S(\vec{r}_1, \vec{r}_2, \vec{r}_3) &= \frac{1}{\sqrt{6}}[\psi_n(\vec{r}_1)\psi_m(\vec{r}_2)\psi_l(\vec{r}_3) + \psi_n(\vec{r}_1)\psi_l(\vec{r}_2)\psi_m(\vec{r}_3) \\ &\quad + \psi_l(\vec{r}_1)\psi_m(\vec{r}_2)\psi_n(\vec{r}_3) + \psi_m(\vec{r}_1)\psi_n(\vec{r}_2)\psi_l(\vec{r}_3) \\ &\quad + \psi_l(\vec{r}_1)\psi_n(\vec{r}_2)\psi_m(\vec{r}_3) + \psi_m(\vec{r}_1)\psi_l(\vec{r}_2)\psi_n(\vec{r}_3)], \quad (n \neq m \neq l)\end{aligned}\quad (37)$$

$$\begin{aligned}\psi_3^S(\vec{r}_1, \vec{r}_2, \vec{r}_3) &= \frac{1}{\sqrt{3}}[\psi_n(\vec{r}_1)\psi_n(\vec{r}_2)\psi_l(\vec{r}_3) + \psi_n(\vec{r}_1)\psi_m(\vec{r}_2)\psi_n(\vec{r}_3) \\ &\quad + \psi_l(\vec{r}_1)\psi_n(\vec{r}_2)\psi_n(\vec{r}_3)], \quad (n = m \neq l)\end{aligned}\quad (38)$$

(2) The antisymmetrical space wave functions of three-electron

$$\psi_1^A(\vec{r}_1, \vec{r}_2, \vec{r}_3) = \frac{1}{\sqrt{3!}} \begin{vmatrix} \psi_n(\vec{r}_1) & \psi_n(\vec{r}_2) & \psi_n(\vec{r}_3) \\ \psi_m(\vec{r}_1) & \psi_m(\vec{r}_2) & \psi_m(\vec{r}_3) \\ \psi_l(\vec{r}_1) & \psi_l(\vec{r}_2) & \psi_l(\vec{r}_3) \end{vmatrix}. \quad (39)$$

Where $\psi_m(\vec{r}_1)$, $\psi_n(\vec{r}_2)$ and $\psi_l(\vec{r}_3)$ are single electron wave function, and m , n and l express quantum numbers of quantum state.

5 Three-electron total wave functions and entanglement states

The indistinguishability principle applied for fermions leads to the multi-fermion total wave functions antisymmetry, i.e., the space wave function is symmetrical, the spin wave function is antisymmetrical, or the space wave function is antisymmetrical, the spin wave function is symmetrical. The three-electron total wave functions are

(a) When $m = n = l$, the three-electron total antisymmetrical wave function

$$\Psi_1^A(q_1, q_2, q_3) = \psi_1^S(r_1, r_2, r_3)\chi_{S'SM}^A(s_{1z}, s_{2z}, s_{3z}), \quad (40)$$

(b) When $m \neq n \neq l$, the three-electron total antisymmetrical wave function

$$\Psi_2^A(q_1, q_2, q_3) = \psi_2^S(r_1, r_2, r_3)\chi_{S'SM}^A(s_{1z}, s_{2z}, s_{3z}), \quad (41)$$

(c) When $m = n \neq l$, the three-electron total antisymmetrical wave function

$$\begin{aligned} \Psi_3^A(q_1, q_2, q_3) = & \frac{1}{\sqrt{3}} [\psi_n(\vec{r}_1)\psi_n(\vec{r}_2)\chi^A(1, 2)\psi_l(\vec{r}_3)\chi(3) + \psi_n(\vec{r}_1)\psi_n(\vec{r}_3)\chi^A(1, 3)\psi_l(\vec{r}_2)\chi(2)) \\ & + \psi_n(\vec{r}_2)\psi_n(\vec{r}_3)\chi^A(2, 3)\psi_l(\vec{r}_1)\chi(1)], \end{aligned} \quad (42)$$

(d) When $m \neq n \neq l$, the three-electron total antisymmetrical wave function

$$\Psi^A(q_1, q_2, q_3) = \psi^A(r_1, r_2, r_3)\chi_{S'SM}^S(s_{1z}, s_{2z}, s_{3z}), \quad (43)$$

where q_1, q_2 and q_3 express the space coordinate and spin component of electron 1, 2 and 3, respectively, the wave functions $\psi^S(r_1, r_2, r_3)$ and $\psi^A(r_1, r_2, r_3)$ are symmetrical and antisymmetrical space wave functions of three-electron, which are shown in equations (36)-(39), the wave functions $\chi_{S'SM}^A(s_{1z}, s_{2z}, s_{3z})$ and $\chi_{S'SM}^S(s_{1z}, s_{2z}, s_{3z})$ are symmetrical and antisymmetrical spin wave functions of three-electron, which are shown in section 3. The equations (40)-(43) are the total wave functions of three-electron, i.e., three-electron full entanglement states, and we can obtain the results: (1) The three-electron entanglement states is related to their space and spin wave functions, and we should not only consider their spin wave function parts. (2) when there are at least one entanglement state for the space wave function or spin wave function, the total wave functions of three-electron are entanglement state. (3) when the space wave function and spin wave function are not entanglement state, the total wave functions of three-electron is not on entanglement state. (4) When the space wave function is entanglement, and the spin wave function is not entanglement, the three-electron is called the space entanglement. (5) When the spin wave function is entanglement, and the space wave function is not entanglement, the three-electron is called the spin entanglement. (6) When the space and spin wave function are all entanglement, the three-electron is called the full entanglement. (7) When the space and spin wave function are not entanglement, the three-electron is not entanglement.

When three electrons are on bound state, whether they are in well potential or central force field and so on, every electron space wave function $\psi_m(\vec{r}) \rightarrow 0$ when $r \geq a_0$ (a_0 is a teeny number). The symmetrical and antisymmetrical space wave functions $\Psi^S(q_1, q_2, q_3)$ and $\Psi^A(q_1, q_2, q_3)$ of three-electron should be tended to zero, then three-electron space entanglement should be broken down and become without entanglement for the three-electron. In experiments [16, 17], the authors have found multiparticle restricted to one entanglement bit rather than an arbitrary amount of entanglement, which are agreement with our theory analysis results.

6 Conclusion

In this paper, we have given the symmetrical and antisymmetrical spin and space wave functions of three-electron, and further given the full total entanglement states for the three-electron, which are related to their space and spin wave function. When we study particles entanglement we not only consider their spin entanglement and also consider their space entanglement. Otherwise, we find that electrons entanglement are restricted to the teeny range, when electrons exceed the space range, their entanglement should be broken down even disappearance, which is accordance with the experiments results.

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